

INTRODUCTION TO COORDINATE GEOMETRY

Define Coordinate Geometry

The study of geometrical shapes in a plane is called plane geometry. Coordinate geometry is the study of geometrical shapes in the Cartesian plane (coordinate plane).

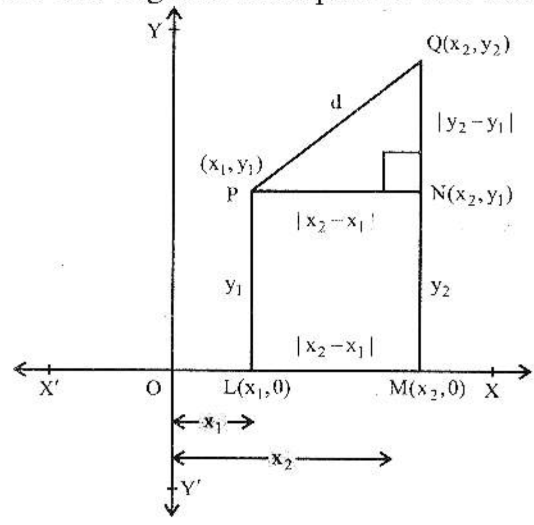
We known that a plane is divided into four quadrants by two perpendicular lines called the axes intersecting at origin. We have also seen that there is one to one correspondence between the points of the plane and the ordered pairs in R × R.

Finding Distance between two points

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be two points in the coordinate plane where d is the length of the line segment PQ. i.e. |PQ| = d.

The line segments MQ and LP parallel to y–axis meet x-axis at points M and L, respectively with coordinates $M(x_2, 0)$ and $L(x_1, 0)$.

The line–segment PN is parallel to x–axis.



In the right triangle PNQ,

$|QN| = |y_2 - y_1|$ and $|PN| = |x_2 - x_1|$.

Using Pythagoras Theorem

$$(\overline{PQ})^2 = (\overline{PN})^2 = (\overline{QN})^2$$

$$\Rightarrow \quad d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$\Rightarrow \quad d = \pm\sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

since $d > 0$ always.

Example

Using the distance formula, find the distance between the points.

(i) P(1, 2) and Q(0,3)

(ii) S(−1, 3) and R(3, −2)

Solutions

(i) $|PQ| = \sqrt{(0-1)^2 + (3-2)^2}$

$= \sqrt{(-1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}$

(ii) $|SR| = \sqrt{(3-(-1))^2 + (-2-3)^2}$

$= \sqrt{(3+1)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$

Collinear or Non-collinear Points in the Plane

Two or more than two points which lie on the same straight line are called collinear points with respect to that line; otherwise they are called non-collinear.

Let PQ be a line, then all the points on line m are collinear.

In the given figure the points P and Q are collinear with respect to the line m and the points P and R are not collinear with respect to it.



Use of Distance Formula to show the Collinearity of Three or more Points in the Plane

Let P, Q and R be three points in the plane. They are called collinear

If $|PQ| + |QR| = |PR|$, otherwise they are non-collinear.

Example

Using distance formula show that the points.

(i) P(−2,−1), Q(0, 3) and R(1, 5) are collinear.

(ii) The above P,Q,R and S (1,−1) are not collinear

Sol. By using the distance formula, we find

$$|PQ| = \sqrt{(0+2)^2 + (3+1)^2}$$
$$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$
$$|QR| = \sqrt{(1-0)^2 + (+5-3)^2}$$
$$= \sqrt{1+4} = \sqrt{5}$$
$$|PR| = \sqrt{(1+2)^2 + (5+1)^2}$$
$$= \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

Since $|PQ| + |QR| = 2\sqrt{5} + \sqrt{5}$
$$= 3\sqrt{5} = |PR|$$

points P, Q, R are collinear.

(ii) The above points P,Q,R and S (1,-1) are not collinear

Sol $|PS| = \sqrt{(-2-1)^2 + (-1+1)^2}$
$$= \sqrt{(-3)^2 + 0} = 3$$

Since $|QS| = \sqrt{(1-0)^2 + (-1-3)^2}$
$$= \sqrt{1+16} = \sqrt{17},$$

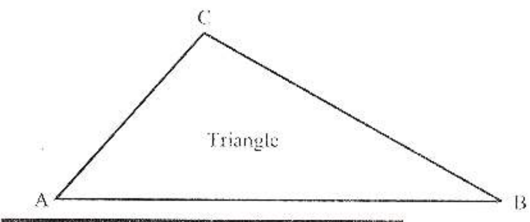
and $|PQ| + |QS| \neq |PS|$,

Therefore the points, P,Q and S are not collinear and hence, the points P, Q, R and S are also not collinear.

Define Triangle

A closed figure in a plane obtained by joining three non-collinear points is called a triangle.

In the triangle ABC the non-collinear points A, B and C are the three vertices of the triangle ABC. The line segments AB, BC and CA are called sides of the triangle.



Define Equilateral Triangle

If the lengths of all the three sides of a triangle are same, then the triangle is called an equilateral triangle.

Example

The triangle OPQ is an equilateral triangle since the points O(0,0), $P\left(\frac{1}{\sqrt{2}}, 0\right)$ and $Q\left(\frac{1}{2\sqrt{2}}, \frac{\sqrt{3}}{2\sqrt{2}}\right)$ are not collinear , where

$$|OP| = \frac{1}{\sqrt{2}}$$

$$|QO| = \sqrt{\left(0-\frac{1}{2\sqrt{2}}\right)^2+\left(0-\frac{\sqrt{3}}{2\sqrt{2}}\right)^2}$$

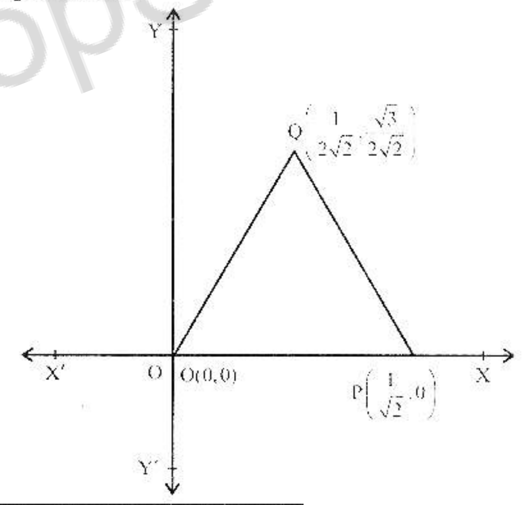
$$=\sqrt{\frac{1}{8}+\frac{3}{8}}=\sqrt{\frac{4}{8}}=\sqrt{\frac{1}{2}}$$

$$|PQ|=\sqrt{\left(\frac{1}{2\sqrt{2}}-\frac{1}{\sqrt{2}}\right)^2+\left(\frac{\sqrt{3}}{2\sqrt{2}}-0\right)^2}$$

$$=\sqrt{\left(\frac{1-2}{2\sqrt{2}}\right)^2+\left(\frac{\sqrt{3}}{2\sqrt{2}}\right)^2}$$

$$=\sqrt{\frac{1}{8}+\frac{3}{8}}=\sqrt{\frac{4}{8}}=\sqrt{\frac{1}{2}}$$

i.e., $|OP|=|QO|=|PQ|=\frac{1}{\sqrt{2}}$, a real number and the points O(0,0), $Q\left(\frac{1}{2\sqrt{2}},\frac{\sqrt{3}}{2\sqrt{2}}\right)$ and $P\left(\frac{1}{\sqrt{2}},0\right)$ are not collinear. Hence the triangle OPQ is equilateral.

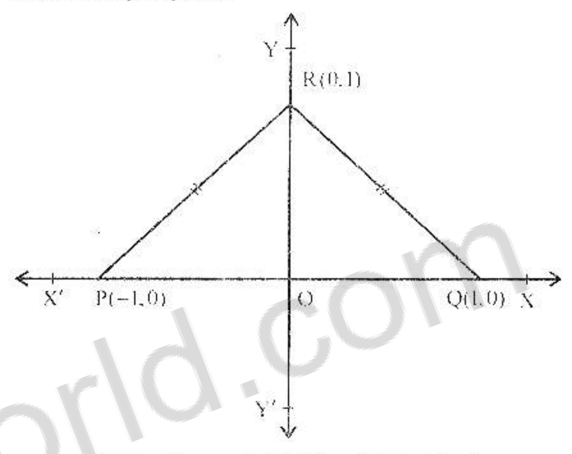


An Isosceles Triangle

An isosceles triangle PQR is a triangle which has two of its sides with equal length while the third side has a different length.

Example

The triangle PQR is an isosceles triangle as for the non-collinear points P(−1,0), Q(1, 0) and R(0, 1) shown in the following figure.



$|PQ|=\sqrt{(1-(-1))^2+(0-0)^2}=\sqrt{(1+1)^2+0}=\sqrt{4}=2$

$|QR|=\sqrt{(0-1)^2+(1-0)^2}=\sqrt{(-1)^2+1^2}=\sqrt{1+1}=\sqrt{2}$

$|PR|=\sqrt{(0-(-1))^2+(1-0)^2}=\sqrt{1+1}=\sqrt{2}$

Since $|QR|=|PR|=\sqrt{2}$ and $|PQ| = 2 \neq \sqrt{2}$

so the non-collinear points P, Q, R form an isosceles triangle PQR.

Right Angle Triangle

A triangle in which one of the angles has measure equal to 90° is called a right angle triangle.

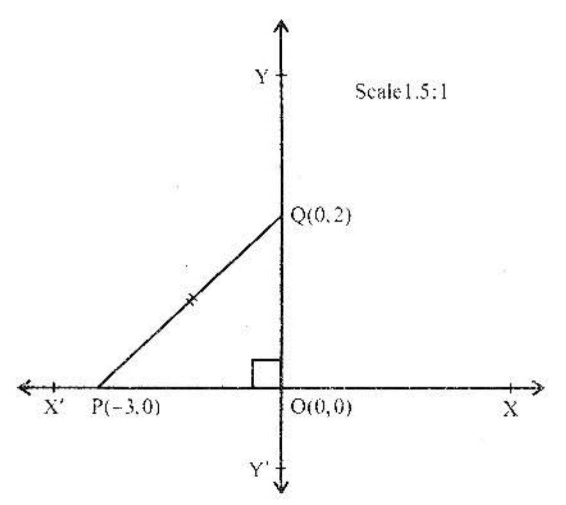
Example

Let O(0, 0), P(−3, 0) and Q(0, 2) be three non-collinear points. Verify that triangle OPQ is right-angled.

$|OQ|=\sqrt{(0-0)^2+(2-0)^2}=\sqrt{2^2}=2$

$|OP|=\sqrt{(-3)^2+0^2}=\sqrt{9}=3$

$|PQ| = \sqrt{(-3)^2+(-2)^2} = \sqrt{9+4}=\sqrt{13}$



Now

$|OQ|^2 + |OP|^2 = (2)^2 + (3)^2 = 13$ and $|PQ|^2 = 13$

Since

$|OQ|^2 + |OP|^2 = |PQ|^2$, therefore $\angle POQ = 90^o$

Hence the given non-collinear points form a right triangle.

Scalene Triangle

A triangle is called a scalene triangle if measures of all the three sides are different.

Example

Show that the points P(1, 2), Q(−2, 1) and R(2, 1) in the plane form a scalene triangle.

Solution

$|PQ| = \sqrt{(-2-1)^2 + (1-2)^2}$

$= \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

$|QR| = \sqrt{(2+2)^2 + (1-1)^2}$

$= \sqrt{4^2 + 0^2} \quad = \sqrt{4^2} = 4$

and

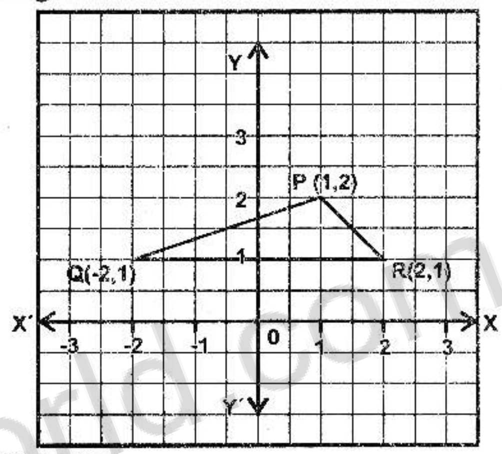
$|PR| = \sqrt{(2-1)^2 + (1-2)^2}$

$= \sqrt{1^2 + (-1)^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$

Hence $|PQ| = \sqrt{10}, |QR| = 4$ and $|PR| = \sqrt{2}$

The points P, Q and R are non-collinear since, $|PQ| + |QR| > |PR|$

Thus the given points form a scalene triangle.



Example

If A(2, 2), B(2, −2), C(−2, −2) and D(−2, 2) be four non-collinear points in the plane, then verify that they form a square ABCD.

Solution

Since $|AB| = \sqrt{(2-2)^2 + (-2-2)^2}$

$= \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$

$|BC| = \sqrt{(-2-2)^2 + (-2+2)^2}$

$= \sqrt{(-4)^2 + 0^2} = \sqrt{16} = 4$

$|CD| = \sqrt{(-2-(-2))^2 + (2-(-2))^2}$

$= \sqrt{(-2+2)^2 + (2+2)^2}$

$= \sqrt{0+16} = \sqrt{16} = 4$

$|DA| = \sqrt{(2+2)^2 + (2-2)^2}$

$= \sqrt{(+4)^2 + 0} = \sqrt{16} = 4$

Hence $|AB| = |BC| = |CD| = |DA| = 4$

Also $|AC|=\sqrt{(-2-2)^2+(-2-2)^2}$

$=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$

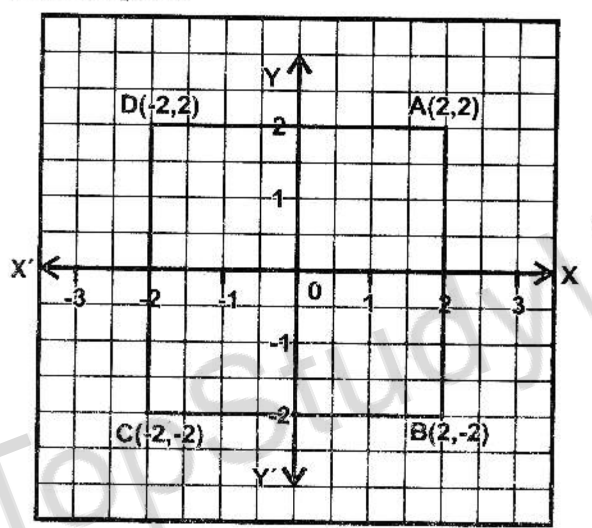
Now $|AB|^2+|BC|^2=(4)^2+(4)^2=32$, and

$|AC|^2=(4\sqrt{2})^2=32$

Since $|AB|^2+|BC|^2=|AC|^2$,

therefore $\angle ABC=90^\circ$

Hence the given four non-collinear points form a square.



Define Parallelogram

A figure formed by four non-collinear points in the plane is called a parallelogram if

(i) its opposite sides are of equal length

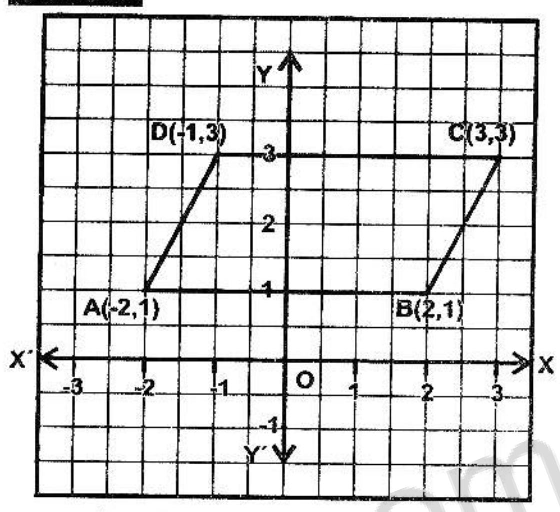
(ii) its opposite sides are parallel

(iii) measure of none of the angles in 90°

Example

Show that the points A(−2, 1), B(2, 1), C(3, 3) and D(−1, 3) form a parallelogram.

Solution:



By distance formula,

$|AB|=\sqrt{(2+2)^2+(1-1)^2}$

$=\sqrt{4^2+0}=\sqrt{16}=\quad 4$

$|CD|=\sqrt{(3+1)^2+(3-3)^2}$

$=\sqrt{4^2+0}=\sqrt{16}=4$

$|AD|=\sqrt{(-1+2)^2+(3-1)^2}$

$=\sqrt{1^2+2^2}=\sqrt{1+4}=\sqrt{5}$

$|BC|=\sqrt{(3-2)^2+(3-1)^2}$

$=\sqrt{1^2+2^2}=\sqrt{5}$

Since

$|AB|=|CD|=4$ and $|AD|=|BC|=\sqrt{5}$

So opposite sides of the quadrilateral ABCD are equal.

Also $|AC|=\sqrt{(3+2)^2+(3-1)^2}$

$=\sqrt{(5)^2+2^2}=\sqrt{25+4}=\sqrt{29}$

Now

$|AB|^2+|BC|^2=16+5=21$ and $|AC|^2=29$

Since in triangle

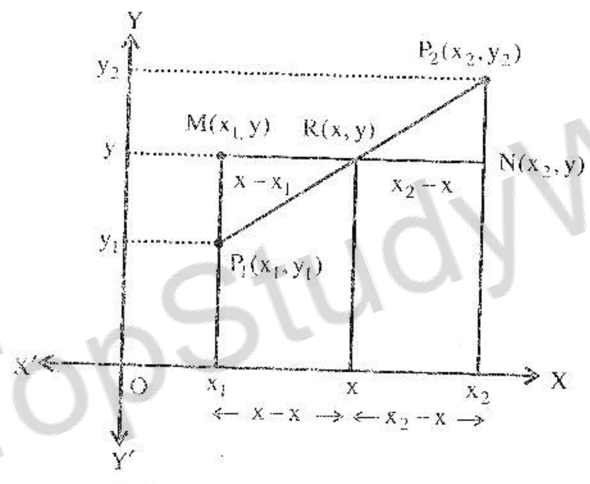
$ABC, |AB|^2 + |BC|^2 \neq |AC|^2$

Therefore measure of angle at $B \neq 90^\circ$

Hence the given points form a parallelogram.

Recognition of the Mid-Point Formula for any two Points in the Plane

Let $P_1(x_1, y_1)$ and $P_2(x_1, y_1)$ be any two points in the plane and $R(x, y)$ be a mid-point of points P_1 and P_2 on the line-segment P_1P_2 as shown in the figure below.



If line-segment MN, parallel to x-axis, has its mid-point $R(x,y)$,

then , $x_2 - x = x - x_1$

$$\Rightarrow \quad 2x = x_1 + x_2 \Rightarrow x = \frac{x_1 + x_2}{2}$$

Similarly, $y = \dfrac{y_1 + y_2}{2}$

Thus the point $R(x, y) = R\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$ is the mid-point of the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$.

Example

Find the mid-point of the line segment joining A(2,5) and B(−1, 1).

Solution

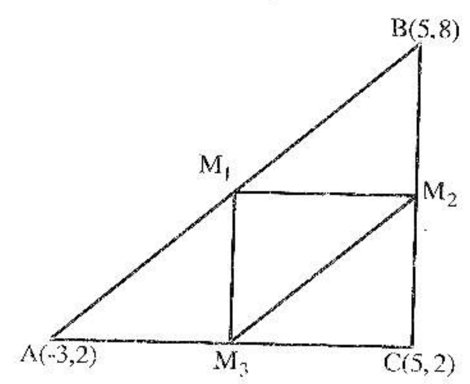
If $R(x, y)$ is the desired mid–point then.

$$x = \frac{2-1}{2} = \frac{1}{2} \text{ and } y = \frac{5+1}{2} = \frac{6}{2} = 3$$

Hence $R(x, y) = R\left(\dfrac{1}{2}, 3\right)$

xample

Let ABC be a triangle as shown below. If M_1, M_2 and M_3 are the middle points of the line-segments AB, BC and CA respectively, find the coordinates of M_1, M_2 and M_3. Also determine the type of the triangle $M_1M_2M_2$.



Solution

Midpoint of $AB = M_1\left(\dfrac{-3+5}{2}, \dfrac{2+8}{2}\right) = M_1(1,5)$

Midpoint of $BC = M_2\left(\dfrac{5+5}{2}, \dfrac{8+2}{2}\right) = M_2(5,5)$

and Mid–point of

$AC = M_3\left(\dfrac{5-3}{2}, \dfrac{2+2}{2}\right) = M_3(1,2)$

The triangle $M_1M_2M_3$ has sides with length,

$$|M_1M_2| = \sqrt{(5-1)^2+(5-5)^2}$$

$$= \sqrt{4^2+0} = 4 \text{ .....}(i)$$

$$|M_2M_3| = \sqrt{(1-5)^2+(2-5)^2}$$

$$= \sqrt{(-4)^2+(-3)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \quad \text{.....}(ii)$$

and $$|M_1M_3| = \sqrt{(1-1)^2+(2-5)^2}$$

$$= \sqrt{0^2+(-3)^2} = 3 \text{.....}(iii)$$

All the lengths of the three sides are different. Hence the triangle $M_1M_2M_3$ is a Scalene triangle.

Example

Let O(0,0), A(3,0) and B(3,5) be three points in the plane. If M_1 is the mid point of AB and M_2 of OB, then show that $|M_1M_2| = \frac{1}{2}|OA|$.

Solution

By the distance formula the distance

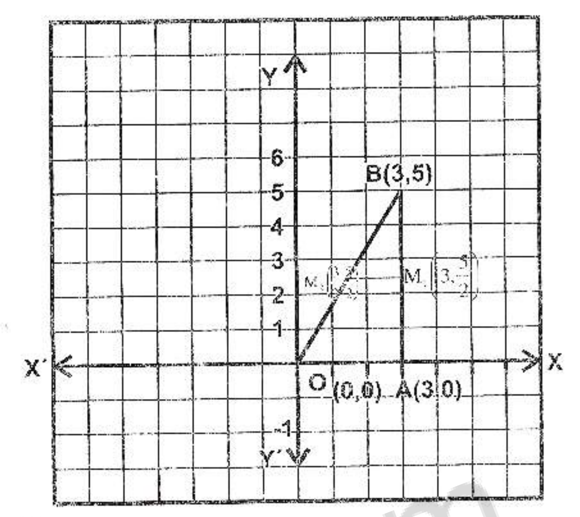
$$|OA| = \sqrt{(3-0)^2+(0-0)^2} = \sqrt{3^2} = 3$$

The mid–point of AB is:

$$M_1 = M_1\left(\frac{3+3}{2}, \frac{5+0}{2}\right) = \left(3, \frac{5}{2}\right)$$

The mid–point of OB is:

$$M_2 = M_2\left(\frac{3+0}{2}, \frac{5+0}{2}\right) = \left(\frac{3}{2}, \frac{5}{2}\right)$$



Hence

$$|M_1M_2| = \sqrt{\left(\frac{3}{2}-3\right)^2+\left(\frac{5}{2}-\frac{5}{2}\right)^2}$$

$$= \sqrt{\left(\frac{-3}{2}\right)^2+0} = \sqrt{\frac{9}{4}+0} = \frac{3}{2}$$

$$= \frac{1}{2}|OA|$$

Note:

Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two points and their midpoint be:

$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. Then M

(i) is at equal distance from P and Q i.e., $|PM| = |MQ|$

(ii) is an interior point of the line segment PQ.

(iii) every point R in the plane at equal distance from P and Q is not their mid point. For example, the point R(0,1) is at equal distance from P(–3, 0) and Q(3, 0) but is not their mid–point.

i.e., $|RQ| = \sqrt{(0-3)^2+(1-0)^2}$

$=\sqrt{(-3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}$

$|RP|=\sqrt{(0+3)^2+(1-0)^2}$

$=\sqrt{3^2+1^2}=\sqrt{10}$

And midpoint of P(–3,0) and Q(3, 0) is

Where $x=\frac{-3+3}{2}=0$

$y=\frac{0+0}{2}=0$

The point (0, 1) ≠ (0, 0).

(iv) There is a unique midpoint of any two points in the plane.

Exercise 9.1

Q1. Find the distance between the following pairs of points

a) $A(9,2), B(7,2)$

Sol. $|AB|=\sqrt{(x_2-x_1)^2+(y_2+y_1)^2}$

$=\sqrt{(7-9)^2+(2-2)^2}$

$=\sqrt{(-2)^2+(0)^2}$

$=\sqrt{4}$

$=2$

b) $A(2,-6), B(3,-6)$

Sol. $|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(3-2)^2+(-6+6)^2}$

$=\sqrt{(1)^2+(0)^2}$

$=\sqrt{1}$

$=1$

c) $A(-8,1), B(6,1)$

Sol. $|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(6+8)^2+(1-1)^2}$

$=\sqrt{(14)^2+(0)^2}$

$|AB|=14$

d) $A(-4,\sqrt{2}), B(-4,-3)$

Sol. $|AB|=\sqrt{(x_2-x_1)^2+(y_2-y)^2}$

$=\sqrt{(-4+4)^2+(-3-\sqrt{2})^2}$

$=\sqrt{(0)^2+(-3-\sqrt{2})^2}$

$=\sqrt{(-3-\sqrt{2})^2}$

$=(3+\sqrt{2})^2$

$=3+\sqrt{2}$

(e) $A(3,-11), B(3,-4)$

Sol. $|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$=\sqrt{(3-3)^2+(-4-(-11))^2}$

$=\sqrt{(0)^2+(7)^2}=\sqrt{(7)^2}=7$

(f) A(0,0) B(0,-5)

$|AB|=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Sol. $=\sqrt{(0-0)^2+(-5-0)^2}$

$=\sqrt{0+(-5)^2}=\sqrt{(5)^2}=5$

Q2. Let P be the point on x-axis with x-coordinate a and Q be the point on y-axis with y-coordinate b, as given below. Find distance between P and Q.

i) $a=9,\ b=7$

$$|PQ|=\sqrt{(9)^2+(7)^2}=\sqrt{81+49}=\sqrt{130}$$

ii) $a=2,\ b=3$

$$|PQ|=\sqrt{(2)^2+(3)^2}=\sqrt{4+9}=\sqrt{13}$$

iii) $a=-8,\ b=6$

$$|PQ|=\sqrt{(-8)^2+(6)^2}=\sqrt{64+36}=\sqrt{100}=10$$

iv) $a=-2,\ b=-3$

$$|PQ|=\sqrt{(-2)^2+(-3)^2}=\sqrt{4+9}=\sqrt{13}$$

v) $a=\sqrt{2},\ b=1$

$$|PQ|=\sqrt{\left(\sqrt{2}\right)^2+(1)^2}=\sqrt{2+1}=\sqrt{3}$$

vi) $a=-9,\ b=-4$

$$|PQ|=\sqrt{(-9)^2+(-4)^2}=\sqrt{81+16}=\sqrt{97}$$

Exercise 9.2

Q1. Show whether the points with vertices $(5,-2),(5,4)$ and $(-4,1)$ are vertices of an equilateral triangle or an isosceles triangle?

SOL. Let P(5,-2) , Q(5,4), R(-4,1)

$$|PQ|=\sqrt{(5-5)^2+(4+2)^2}=\sqrt{(0)^2+(6)^2}=\sqrt{36}=6$$

$$|QR|=\sqrt{(-4-5)^2+(1-4)^2}=\sqrt{81+9}=\sqrt{90}$$

$$|PR|=\sqrt{(-4-5)^2+(1+2)^2}=\sqrt{81+9}=\sqrt{90}$$

Since $|QR|=|PR|=\sqrt{90}$ and

$|PQ|=6\neq\sqrt{90}$

So the non collinear points P,Q,R form an isosceles triangle PQR

Q2. Show whether or not the points with vertices $(-1,1),(5,4),(2,-2)$ and $(-4,1)$ form a square.

Sol. Let $A(-1,1),B(5,4),C(2,-2),D(-4,1)$

Since $|AB|=\sqrt{(5+1)^2+(4-1)^2}$

$$=\sqrt{6^2+3^2}=\sqrt{36+9}=\sqrt{45}$$

$$|BC|=\sqrt{(2-5)^2+(-2-4)^2}$$

$$|BC|=\sqrt{(-3)^2+(-6)^2}=\sqrt{9+36}=\sqrt{45}$$

$$|CD|=\sqrt{(-4-2)^2+(1+2)^2}$$

$$=\sqrt{(-6)^2+(3)^2}=\sqrt{36+9}=\sqrt{45}$$

$$|DA|=\sqrt{(-4+1)^2+(1-1)^2}$$

$$=\sqrt{(-3)^2+(0)^2}=\sqrt{9}=3$$

Hence $|AB|=|BC|=|CD|=\sqrt{45}$

but $|DA|\neq\sqrt{45}$

Hence given points do not form a square.

Q3. Show whether or not the points with coordinates $(1,3)(4,2)$, and $(-2,6)$ are vertices of a right triangle.

Sol. Let $P(1,3)$, $Q(4,2)$ and $R(-2,6)$

$$|PQ|=\sqrt{(4-1)^2+(2-3)^2}$$

$$=\sqrt{(3)^2+(1)^2}=\sqrt{9+1}=\sqrt{10}$$

$$|QR| = \sqrt{(-2-4)^2 + (6-2)^2}$$
$$= \sqrt{36+16} = \sqrt{52}$$
$$|PR| = \sqrt{(-2-1)^2 + (6-3)^2}$$
$$|BC| = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

Now $|PQ|^2 + |QR|^2 = \left(\sqrt{10}\right)^2 + \left(\sqrt{52}\right)^2$
$$= 10 + 52 = 62$$

and $|PR|^2 = \left(\sqrt{18}\right)^2 = 18$
$$|PQ|^2 + |QR|^2 \neq |PR|^2$$

So triangle is not right angled

Q4. Use the distance formula to prove whether or not the points $(1,1), (-2,-8)$ and $(4,10)$ lie on a straight line.

Let $A(1,1)$, $B(-2,-8)$, $C(4,10)$

Since $|AB| = \sqrt{(-2-1)^2 + (-8-1)^2}$
$$= \sqrt{(-3)^2 + (-9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$$
$$|BC| = \sqrt{(4+2)^2 + (10+8)^2}$$
$$|BC| = \sqrt{(6)^2 + (18)^2}$$
$$= \sqrt{36+324} = \sqrt{360}$$
$$= \sqrt{2\times2\times2\times3\times3\times5} = 6\sqrt{10}$$
$$|AC| = \sqrt{(4-1)^2 + (10-1)^2}$$
$$= \sqrt{(3)^2 + (9)^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$$
$$|AB| + |AC| = 3\sqrt{10} + 3\sqrt{10}$$
$$= 6\sqrt{10} = |BC|$$

So $|AB| + |AC| = |BC|$ the points A, B and C are collinear.

Q5. Find K given that the point $(2,K)$ is equidistance from $(3,7)$ and $(9,1)$.

Sol. Let $P(2,K), Q(3,7)$ and $R(9,1)$
$$|PQ| = \sqrt{(3-2)^2 + (7-K)^2}$$
$$= \sqrt{1^2 + (7-K)^2} = \sqrt{1 + (7-K)^2}$$
$$= \sqrt{1 + 49 - 2(7)k + k^2}$$
$$= \sqrt{50 - 14k + k^2}$$
$$|PR| = \sqrt{(9-2)^2 + (1-K)^2}$$
$$= \sqrt{49 + 1 - 2(1)k + k^2}$$
$$= \sqrt{50 - 2k + k^2}$$

As point P is equidistant from Q and

So $|PQ| = |PR|$
$$\sqrt{50 - 14k + k^2} = \sqrt{50 - 2k + k^2}$$
$$50 - 14k + k^2 = 50 - 2k + k^2$$
$$-12k = 0 \Rightarrow k = 0$$

Q6. Use distance formula to verify that the points $A(0,7)$, $B(3,-5)$, $C(-2,15)$ are collinear.

So $|AB| = \sqrt{(3-0)^2 + (-5-7)^2}$
$$= \sqrt{9 + (-12)^2} = \sqrt{9+144}$$
$$= \sqrt{153} = 12.37$$
$$|BC| = \sqrt{(-2-3)^2 + (15+5)^2}$$
$$= \sqrt{25+400} = \sqrt{425} = 20.62$$
$$|CA| = \sqrt{(-2-0)^2 + (15-7)^2}$$
$$= \sqrt{4+64} = \sqrt{68} = 8.25$$

As $|AB| + |CA| = |BC|$

So given points are collinear with A between B and C.

Q7. Verify whether or not the points $O(0,0)$, $A\left(\sqrt{3},1\right), B\left(\sqrt{3}-1\right)$ are vertices of a equilateral triangle.

Sol. $|OA| = \sqrt{(\sqrt{3}-0)+(1-0)^2}$

$= \sqrt{\left(\sqrt{3}\right)^2 + (1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

$|AB| = \sqrt{\left(\sqrt{3}-\sqrt{3}\right)^2 + (-1-1)^2}$

$= \sqrt{(0)^2 + (-2)^2} = \sqrt{0+4} = 2$

$|OB| = \sqrt{\left(\sqrt{3}-0\right)^2 + (-1-0)^2}$

$= \sqrt{\left(\sqrt{3}\right)^2 + (-1)^2}$

$= \sqrt{3+1} = \sqrt{4} = 2$

As $|OA| = |AB| = |OB| = 2$

Hence points are not collinear.

$\therefore$ the triangle OAB is equilateral

Q8. Show that the points A(-6,-5), B(5,-5), C(5,-8), D(-6,-8) are vertices of a rectangle. Find the lengths of its diagonals. Are they equal ?.

Sol. $|AB| = \sqrt{(5+6)^2 + (-5+5)^2}$

$= \sqrt{(11)^2 + (0)^2} = \sqrt{121} = 11$

$|BC| = \sqrt{(5-5)^2 + (-8+5)^2}$

$= \sqrt{(0)^2 + (-3)^2} = \sqrt{9} = 3$

$|DC| = \sqrt{(5+6)^2 + (-8+8)^2}$

$= \sqrt{(11)^2 + (0)^2} = \sqrt{121} = 11$

$|AD| = \sqrt{(-6+6)^2 + (-8+5)^2}$

$= \sqrt{(-3)^2} = \sqrt{9} = 3$

Since $|AB| = |DC| = 11$ and

$|AD| = |BC| = 3$ opposite sides are equal

Diagonal $|AC| = \sqrt{(5+6)^2 + (-8+5)^2}$

$= \sqrt{11^2 + 3^2} = \sqrt{121+9} = \sqrt{130}$

Diagonal $|BD| = \sqrt{(-6-5)^2 + (-8+5)^2}$

$= \sqrt{11^2 + 3^2} = \sqrt{121+9} = \sqrt{130}$

$|AD|^2 + |DC|^2 = |AC|^2$

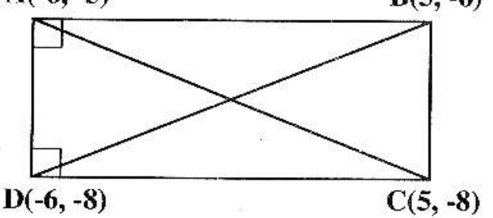
$\therefore \quad \angle ADC = 90^o$

Also $|AB|^2 + |AD|^2 = |BD|^2$

$\therefore \angle BAD = 90^o$

$|AC| = |BD| = \sqrt{130}$

Hence given points form rectangle



As $|AC| = |BD| = \sqrt{130}$

Hence diagonals are equal.

Q9. Show that the points $M(-1,4)$, $N(-5,3), P(1,-3)$ and $Q(5,-2)$ are the vertices of a parallelogram.

SOL. $|PQ| = \sqrt{(5-1)^2 + (-2+3)^2}$

$= \sqrt{(4)^2 + (1)^2} = \sqrt{16+1} = \sqrt{17}$

$$|MN|=\sqrt{(-5+1)^2+(3-4)^2}$$
$$=\sqrt{(-4)^2+(-1)^2}=\sqrt{16+1}=\sqrt{17}$$
$$|NP|=\sqrt{(1+5)^2+(-3-3)^2}$$
$$=\sqrt{(6)^2+(-6)^2}=\sqrt{36+36}=\sqrt{72}$$
$$|MQ|=\sqrt{(5+1)^2+(-2-4)^2}$$
$$=\sqrt{6^2+(-6)^2}=\sqrt{36+36}=\sqrt{72}$$

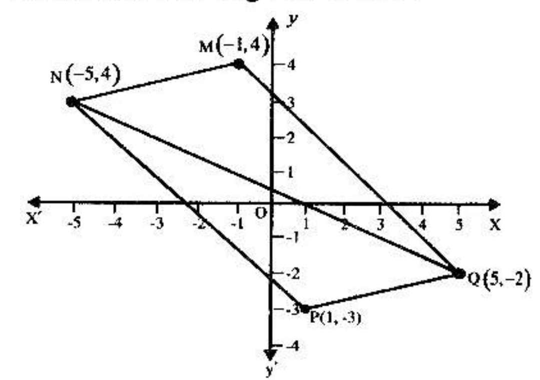
Since $|PQ|=|MN|=\sqrt{17}$

and $|NP|=|MQ|=\sqrt{72}$

So opposite sides, of quadrilateral MNPQ are equal.

$$|NQ|=\sqrt{(-5-5)^2+(3+2)^2}$$
$$=\sqrt{(-10)^2+(5)^2}$$
$$=\sqrt{100+25}=\sqrt{125}=5\sqrt{5}$$
$$|PN|^2+|PQ|^2=\left(\sqrt{72}\right)^2+\left(\sqrt{17}\right)^2$$
$$=72+17=89$$
$$|PN|^2+|PQ|^2\neq|NQ|^2$$

The measure of angle at $P\neq 90^o$



Hence given points form a parallelogram.

Q10. Find the length of the diameter of the circle having centre at $C(-3,6)$ and passing through $P(1,3)$.

SOL. Length of radius=

$$|PC|=\sqrt{(-3-1)^2+(6-3)^2}$$
$$=\sqrt{(-4)^2+(3)^2}$$
$$=\sqrt{16+9}$$
$$=\sqrt{25}$$
$$=5$$

Length of diameter $=2r=2(r)=10$

Exercise 9.3

Q1. Find the mid point of the line segment joining each of the following pairs of points.

a) $A(9,2)$, $B(7,2)$

If $R(x,y)$ is the desired midpoint then,

$$x=\frac{x_1+x_2}{2}=\frac{9+7}{2}=\frac{16}{2}=8$$
$$y=\frac{y_1+y_2}{2}=\frac{2+2}{2}=\frac{4}{2}=2$$
$$\therefore\ R(x,y)=R(8,2)$$

b) $A(2,6)$, $B(3,-6)$

If $R(x,y)$ is the desired midpoint then,

$$x=\frac{x_1+x_2}{2}=\frac{2+3}{2}=\frac{5}{2}$$
$$y=\frac{y_1+y_2}{2}=\frac{6-6}{2}=\frac{0}{2}=0$$
$$R(x,y)=R\left(\frac{5}{2},0\right)$$

c) $A(-8,1)$, $B(6,1)$

If $R(x,y)$ is the desired midpoint then

$$x=\frac{x_1+x_2}{2}=\frac{-8+6}{2}=\frac{-2}{2}=-1$$

$$y=\frac{y_1+y_2}{2}=\frac{1+1}{2}=\frac{2}{2}=1$$

$$\therefore\ R(x,y)=R(-1,1)$$

d) $A(-4,9)$, $B(-4,-3)$

If $R(x,y)$ is the desired mid point then,

$$x=\frac{x_1+x_2}{2}=\frac{-4-4}{2}=\frac{-8}{2}=-4$$

$$y=\frac{y_1+y_2}{2}=\frac{9-3}{2}=\frac{6}{2}=3$$

$$R(x,y)=R(-4,3)$$

e) $A(3,-11)$, $B(3,-4)$

If $R(x,y)$ is the desired midpoint then,

$$x=\frac{x_1+x_2}{2}=\frac{3+3}{2}=\frac{6}{2}=3$$

$$y=\frac{y_1+y_2}{2}=\frac{-11-4}{2}=\frac{-15}{2}=-7.5$$

$$\therefore\ R(x,y)=R(3,-7.5)$$

f) $A(0,0)$, $B(0,-5)$

If $R(x,y)$ is the desired midpoint then,

$$x=\frac{x_1+x_2}{2}=\frac{0+0}{2}=0$$

$$y=\frac{y_1+y_2}{2}=\frac{0-5}{2}=\frac{-5}{2}=-2.5$$

$$\therefore\ R(x,y)=R(0,-2.5)$$

Q2. The end point P of a line segment PQ (–3,6) and its mid point is (5,8). Find the co-ordinates of the end point Q.

Sol: $(-3,6)$

If $R(x,y)$ is mid point then,

$$x=\frac{x_1+x_2}{2}\Rightarrow 5=\frac{-3+x_2}{2}$$

$$\Rightarrow 10=-3+x_2$$

$$x_2=10+3=13$$

and $y=\frac{y_1+y_2}{2}\Rightarrow 8=\frac{6+y_2}{2}$

$$\Rightarrow 16=6+y_2$$

$$y_2=10$$

$\therefore$ Coordinates of the end point $Q(13,10)$

Q3. Prove that midpoint of the hypotenuse of a right triangle is equidistant from its three vertices $P(-2,5)$, $Q(1,3)$ and $R(-1,0)$

SOL. $|PQ|^2=(1+2)^2+(3-5)^2=9+4=13$

$$|QR|^2=(-1-1)^2+(0-3)^2=4+9=13$$

$$|PR|^2=(-1+2)^2+(0-5)^2=1+25=26$$

As $|PQ|^2+|QR|^2=|PR|^2$

Hence PR is the hypotenuse

If $M(x,y)$ is desired midpoint then,

$$x=\frac{-1+(-2)}{2}=\frac{-1-2}{2}=\frac{-3}{2}$$

$$y=\frac{5+0}{2}=\frac{5}{2}$$

$$\therefore\quad M(x,y)=M\left(\frac{-3}{2},\frac{5}{2}\right)$$

Now $|PM|=\sqrt{\left(\frac{-3}{2}+2\right)^2+\left(\frac{5}{2}-5\right)^2}$

$$=\sqrt{\left(\frac{-3+4}{2}\right)^2+\left(\frac{5}{2}-10\right)^2}$$

$$=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{-5}{2}\right)^2}$$

$$=\sqrt{\frac{1}{4}+\frac{25}{4}}$$

$$=\sqrt{\frac{26}{4}}$$

$$|RM|=\sqrt{\left(-\frac{3}{2}+1\right)^2+\left(\frac{5}{2}-0\right)^2}$$

$$=\sqrt{\left(\frac{-3+2}{2}\right)^2+\left(\frac{5-0}{2}\right)^2}$$

$$=\sqrt{\left(\frac{-1}{2}\right)^2+\left(\frac{5}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{25}{4}}$$

$$=\sqrt{\frac{1+25}{4}}=\sqrt{\frac{26}{4}}$$

$$|QM|=\sqrt{\left(-\frac{3}{2}-1\right)^2+\left(\frac{5}{2}-3\right)^2}$$

$$=\sqrt{\left(\frac{-3-2}{2}\right)^2+\left(\frac{5-6}{2}\right)^2}$$

$$=\sqrt{\left(\frac{-5}{2}\right)^2+\left(-\frac{1}{2}\right)^2}=\sqrt{\frac{25}{4}+\frac{1}{4}}==\sqrt{\frac{26}{4}}$$

As $|PM|=|RM|=|QM|$

$\therefore$ M is equidistant from P, Q and R.

Q4. O (0, 0), A(3, 0) and B(3, 5) are three points in the plane, find M_1 and M_2 as midpoints of the line segments $\overline{AB}$ and $\overline{OB}$ respectively. Find $|M_1, M_2|$.

Sol: Let O (0,0), A(3,0), B(3,5) are three points in the plane. M_1 is the mid point of $\overline{OB}$ and M_2 is the mid-point of $\overline{AB}$

$$M(x,y)=M\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=M_1\left(\frac{0+3}{2},\frac{0+5}{2}\right)$$

$$=M_1\left(\frac{3}{2},\frac{5}{2}\right)$$

M_2 is midpoint of $\overline{AB}$ therefore

$$M_2\left(\frac{3+3}{2},\frac{0+5}{2}\right)=M_2\left(\frac{6}{2},\frac{5}{2}\right)$$

$$=M_2\left(3,\frac{5}{2}\right)$$

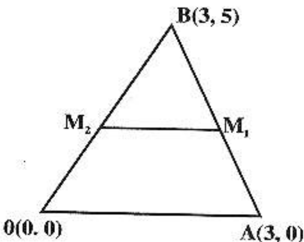
Now $\left(\frac{3}{2},\frac{5}{2}\right)$ and $\left(3,\frac{5}{2}\right)$ are midpoints we find $|M_1 M_2|$

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

Then $|M_1 M_2|=\sqrt{\left(3-\frac{3}{2}\right)^2+\left(\frac{5}{2}-\frac{5}{2}\right)^2}$

$$=\sqrt{\left(\frac{6-3}{2}\right)+0}$$

$$=\sqrt{\left(\frac{3}{2}\right)^2+0}=\frac{3}{2}$$



Q5. Show that the diagonals of the parallelogram having vertices $A(1,2)$, $B(4,2)$, $C(-1,-3)$, $D(-4,-3)$ bisect each other.

Sol: If M_1 is desired midpoint of diagonal DB.

$$x=\frac{x_1+x_2}{2}=\frac{4-4}{2}=0$$

$$y=\frac{y_1+y_2}{2}=\frac{2-3}{2}=\frac{-1}{2}$$

$$M_1(x,y)=\left(0,-\frac{1}{2}\right)$$

If M_2 is desired midpoint of diagonal AC

$$x=\frac{x_1+x_2}{2}=\frac{1-1}{2}=0$$

$$y=\frac{y_1+y_2}{2}=\frac{2-3}{2}=\frac{-1}{2}$$

$$M_2(x,y)=\left(0,-\frac{1}{2}\right)$$

$\therefore$ As midpoints of the diagonals coincide hence diagonal bisect each other.

Q6. The vertices of a triangle are P(4,6), Q(–2,–4) and R(–8, 2) show that the length of line segment joining the mid points of line segment PR, QR is $\frac{1}{2}$ PQ.

Sol. If M_1 is desired midpoint of line segment PR.

$$x=\frac{x_1+x_2}{2}=\frac{4-8}{2}=\frac{-4}{2}=-2$$

$$y=\frac{y_1+y_2}{2}=\frac{6+2}{2}=\frac{8}{2}=4$$

$$M_1(x,y)=M_1(-2,4)$$

If M_2 is desired midpoint of line segment QR.

$$x=\frac{x_1+x_2}{2}=\frac{-2-8}{2}=\frac{-10}{2}=-5$$

$$y=\frac{y_1+y_2}{2}=\frac{-4+2}{2}=\frac{-2}{2}=-1$$

$$M_2(x,y)=M_2(-5,-1)$$

$$|M_1M_2|=\sqrt{(-5+2)^2+(-1-4)^2}$$

$$=\sqrt{(-3)^2+(-5)^2}$$

$$=\sqrt{9+25}=\sqrt{34}$$

$$|PQ|=\sqrt{(-2-4)^2+(-4-6)^2}$$

$$=\sqrt{(-6)^2+(-10)^2}$$

$$=\sqrt{36+100}=\sqrt{136}=\sqrt{34\times 4}$$

$$=2\sqrt{34}$$

As $2|M_1M_2|=|PQ|$

Hence $|M_1M_2|=\frac{1}{2}|PQ|$

Review Exercise 9

Q3. Find distance between pairs of points

i) $(6,3),(3,-3)$

Let $P(6,3),Q(3,-3)$

$$|PQ|=\sqrt{(3-6)^2+(-3-3)^2}$$

$$=\sqrt{(-3)^2+(-6)^2}$$

$$=\sqrt{9+36}=\sqrt{45}$$

ii) $(7,5),(1,-1)$

Let $P(7,5),Q(1,-1)$

$$|PQ|=\sqrt{(7-1)^2+(5+1)^2}$$

$$=\sqrt{(6)^2+(6)^2}=\sqrt{36+36}$$

$$=\sqrt{72}=\sqrt{36\times 2}=6\sqrt{2}$$

iii) $(0,0),(-4,-3)$

Let $P(0,0),Q(-4,3)$

$$|PQ|=\sqrt{(-4-0)^2+(-3-0)^2}$$

$=\sqrt{(-4)^2+(-3)^2}$

$=\sqrt{16+9}=\sqrt{25}=5$

Q4. Find the midpoint between the following pairs of points.

SOL. (i) $(6,6),(4,-2)$

If $R(x,y)$ be desired midpoint, then,

$x=\frac{6+4}{2}=\frac{10}{2}=5$

$y=\frac{6-2}{2}=\frac{4}{2}=2$

$R(x,y)=R(5,2)$

ii) $(-5,-7),(-7,-5)$

If $R(x,y)$ be desired midpoint, then,

$x=\frac{-5-7}{2}=\frac{-12}{2}=-6$

$y=\frac{-5-7}{2}=\frac{-12}{2}=-6$

$\therefore$ $R(x,y)=R(-6,-6)$

iii) $(8,0),(0,-12)$

If $R(x,y)$ be desired midpoint, then,

$x=\frac{8+0}{2}=\frac{8}{2}=4$

$y=\frac{-12+0}{2}=\frac{-12}{2}=-6$

$\therefore$ $R(x,y)=R(4,-6)$

Objective

1. Distance between points (0, 0) and (1, 1) is:

(a) 0 (b) 1

(c) $\sqrt{2}$ (d) 2

2. Distance between the points (1, 0) and (0, 1) is:

(a) 0 (b) 1

(c) $\sqrt{2}$ (d) 2

3. Mid-point of the points (2, 2) and (0,0) is:

(a) (1, 1) (b) (1, 0)

(c) (0, 1) (d) (−1, −1)

4. Mid-point of the points (2, −2) and (−2, 2) is:

(a) (2, 2) (b) (−2, −2)

(c) (0, 0) (d) (1, 1)

5. A triangle having all sides equal is called

(a) Isosceles (b) Scalene

(c) Equilateral (d) None of these

6. A triangle having all sides different is called:

(a) Isosceles (b) Scalene

(c) Equilateral (d) None of these

7. The points P, Q and R are collinear if:

(a) $|PQ|+|QR|=|PR|$

(b) $|PQ|-|QR|=|PR|$

(c) $|PQ|+|QR|=0$

(d) None

8. The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ in the coordinate plane is:

(a) $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$, $d>0$

(b) $d=\sqrt{(x_1-x_2)^2-(y_1-y_2)^2}$

(c) $d=\sqrt{(x_2-x_1)^2-(y_2-y_1)^2}$

(d) $d=\sqrt{(x_1+x_2)^2-(y_1+y_2)^2}$

9. A triangle having two sides equal is called

(a) Isosceles (b) Scalene

(c) Equilateral (d) None

10. A right triangle is that in which one of the angles has measure equal to:

(a) 80° (b) 90°

(c) 45° (d) 60°

11. In a right angle triangle ABC, Pythagoras's theorem,

(a) $|AB|^2=|BC|^2+|CA|^2$ where $\angle ACB=90^\circ$.

(b) $|AB|^2=|BC|^2-|CA|^2$

(c) $|AB|^2+|BC|^2>|CA|^2$

(d) $|AB|^2-|BC|^2>|CA|^2$

Answer key

1.	c	**2.**	c	**3.**	a	**4.**	c	**5.**	c	**6.**	b
7.	a	**8.**	a	**9.**	a	**10.**	b	**11**	a		